

# A New Approach to the Digital Implementation of Analog Controllers for a Power System Control

G. Shabib, Esam H. Abd-Elhameed, G. Magdy

**Abstract**—In this paper a comparison of discrete time PID, PSS controllers is presented through small signal stability of power system comprising of one machine connected to infinite bus system. This comparison achieved by using a new approach of discretization which converts the S-domain model of analog controllers to a Z-domain model to enhance the damping of a single machine power system. The new method utilizes the Plant Input Mapping (PIM) algorithm. The proposed algorithm is stable for any sampling rate, as well as it takes the closed loop characteristic into consideration. On the other hand the traditional discretization methods such as Tustin's method is produce satisfactory results only; when the sampling period is sufficiently low.

**Index Terms**— Dynamic stability, Power systems, PID controllers, Digital redesign, Discretization, Discrete systems.

## 1. INTRODUCTION

The analog control system is a system in which continuous-time controllers are employed to control the behavior of a continuous-time plant. On the other hand, a digital control system includes discrete-time controllers together with a continuous time plant, where the interconnection between digital and analog parts is typically realized through sample and hold devices. For many years, analog controllers have dominated the control of power systems but digital controls have continued to improve in cost and usability in the past several years. This has made digital controls more appealing to replace analog control in some devices. The increasing complexity of power system requires the use of digital devices. They are widely spread and play an essential task in the operation of power systems. Several kinds of digital controlled devices are used in practical in power system now days, such as digital automatic voltage regulator (AVR), digital proportional-integral-derivative (PID) controller and digital power system stabilizer (PSS) [1]. In a distributed power system it would be very helpful to have the controllers alert the operator of a potential problem.

There are several reasons why digital control is desired over analog control. These reasons also apply in converters and in distributed power systems as a whole. The following are citations directly from papers [2], [3], [4] that describe the reasons of using digital control system than analog control system. Potential advantages of digital controller implementation include much improved flexibility, reduced design time, programmability, and elimination of discrete tuning components, improved system reliability, easier system integration and possibility to include various performance enhancements. The opportunity to realize non-linear, predictive and adaptive control strategies provides a strong reason why digital control could yield worthwhile advantages compared with traditional analog control concepts [5].

Some advantages of digital control system are as follows [6]:

- a) Greater range of control algorithms can be used e.g. adaptive control techniques.
- b) Changing a few parameters or implementing a complete new strategy in most cases is just a matter of recompiling a software program model, which on contrary to analog control system that changing components.
- c) Relatively low cost and high computational speeds can be provided with almost any size process.
- d) The ability to interface readily with other computer systems and integration with remote systems.
- e) Easier to implement complicated algorithms.

There are many different approaches to designing discrete-time controllers for a continuous-time system in a feedback configuration. The simplest and most conventional approach involves the approximation of predesigned continuous-time controller transfer functions with discrete-time ones using some particular scheme. This process is therefore suitably called local discretization, since the discretization is applied locally only to the controller without consideration for the overall closed loop system performance. To account for this, Markazi et al. in [7] and Markazi in [8] proposed a discretization method called the Plant input Mapping (PIM) method [9, 10, 11, 12 and 13] which is directly applicable to single input-single output (SISO) as well as multi-input multi-output (MIMO) systems. This method is based on the discretization of the transfer function from the reference input of the closed loop system to the input of the plant which is defined as plant input transfer function (PITF) using the standard Pole-Zero Matching (MPZ) method [10]. Therefore the PIM method, appropriately termed a global digital re-design method takes into account the continuous time closed loop system performance in the formulation of a discrete-time closed loop control system.

The main advantages of the new approach are the guaranteed closed-loop stability for almost all sampling periods, and the recovery of the continuous-time performance when the sampling period approaches zero. The result guarantees closed loop stability for all non-pathological sampling periods [14].

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In the digital redesign technique, a good-designed continuous time controller is converted to a digital controller counterpart. It is based on an optimal matching of continuous-time closed loop step responses of both continuous-time and discretized systems.

In recent years, applications of discrete time controllers to power systems were reported in a number of publications [6, 15 and 16]. It solved the transient stability problem addressed by analog controller, except that discrete time controller is just a matter of reprogramming a software program. [17].

In [6] a technique based on sampled-data control was proposed for optimal discretizations of analog controllers while taking into account both closed-loop and intersample behavior. In [15] a discrete fuzzy PID excitation controller utilizing the bilinear transforms (Tustin's method) was implemented. This controller was developed by first designing discrete time linear PID control law and then progressively driving the steps necessary to incorporate a fuzzy logic control mechanism into the modification of the PID structure. The method in [16] presented a digital redesign method for discretization a continuous-time power system stabilizer PSS for a single machine power system using PIM method. This technique guaranteed the stability for any sampling rate as well as it took closed-loop characteristics into consideration.

In this paper the traditional (Tustin's method) and proposed (PIM method) of discretization methods are applied to digital design of an analog PID, PSS controllers and makes a comparison of discrete-time controllers which is presented through small signal stability of power system comprising of one machine connected to infinite bus and modeled through six K-constants. The components of power system are synchronous machine, exciter, power system stabilizer PSS and PID.

Our goal in this paper is to develop a high performance digital controller for single machine infinite bus power system that takes into consideration the closed loop performance, which cannot be attained when using the traditional digital redesign method. The PIM method is a discretization scheme that can guarantee the stability for any sampling rates (non-pathological sampling rates) [9, 10, 18, and 12].

This paper is organized as follows. In section (2), the standard PIM digital redesign method is considered. Section (3), describes the system configuration that consists of three subsections, which are driving a power system model, explains the continuous time proportional integral and derivative PID controller model and explains the power system stabilizer model. Section (4), application of traditional Discretization method (Tustin's Method) to Power System Model. Section (5), application of Proposed Discretization Method (PIM Method) to Power System Model. Describe the system response and analysis in section (6). Finally the conclusions are given in section (7).

## 2. PLANT INPUT MAPPING METHOD

The Plant Input Mapping (PIM) method is a global digital re-design method for converting an analog control system into a digital counterpart. By taking into account the closed-

loop characteristic of the analog control system in the form of (PITF), the PIM method can guarantee the stability for any nonpathological control rate, has good performances even for low control rates, and is applicable to a variety of analog control methods. Consider the continuous-time plant is linear, time-invariant, and strictly proper, and is denoted as

$$\bar{G}(s) = \frac{\bar{n}_G(s)}{\bar{d}_G(s)} \quad (1)$$

The plant transfer function  $\bar{G}(s)$  is now discretized using the step invariant-model (SIM), which is a combination of the zero-order-hold (ZOH), the plant and the sampler.

Let the step-invariant model of this plant be expressed as

$$G(\varepsilon) = SIM(\bar{G}(s)) = \frac{n_G(\varepsilon)}{d_G(\varepsilon)} \quad (2)$$

The plant is expressed in Euler operator [19], which is defined as

$$\varepsilon = \frac{z-1}{T} \quad (3)$$

Where  $z$  is the usual zee operator and  $T$  is the sampling interval. The Euler operator is used here for better numerical properties in digital control implementation and ease of relating discrete-time results to continuous -time counterparts [12]. Consider the analog control system represented in Fig. 1.

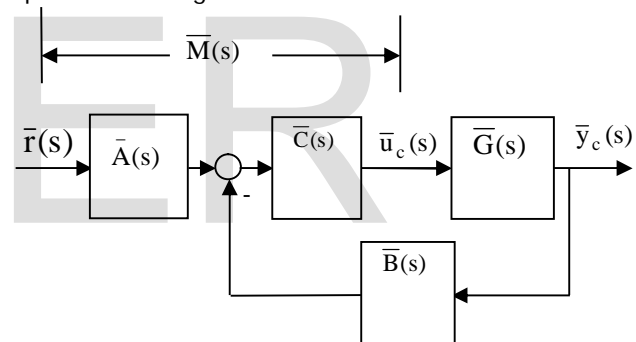


Fig. 1 Continuous-time control system.

Assume that the analog control system is internally stable, satisfies all the design specifications, and is realized with proper transfer functions, which given as;

$$\bar{A}(s) = \frac{\bar{n}_A(s)}{\bar{d}_A(s)}, \quad \bar{B}(s) = \frac{\bar{n}_B(s)}{\bar{d}_B(s)}, \quad \bar{C}(s) = \frac{\bar{n}_C(s)}{\bar{d}_C(s)} \quad (4)$$

In the PIM method, both the closed-loop characteristics and plant information are used in the discretization process in the name of the Plant-Input-Transfer Function (PITF). The PITF is the transfer function from the reference input to the plant input and is given by

$$\bar{M}(s) = \frac{\bar{u}_c(s)}{\bar{r}(s)} = \frac{\bar{A}(s)\bar{C}(s)}{1 + \bar{B}(s)\bar{C}(s)\bar{G}(s)} \quad (5)$$

The PITF is discretized in the standard PIM method. This is carried out using the Matched-pole-zero (MPZ) method [20] and the resulting discrete time model becomes the target PITF. The target discrete-time PITF can be expressed as

$$M(\epsilon) = MPZ(\bar{M}(s)) = \frac{n_M(\epsilon)d_G(\epsilon)}{d_M(\epsilon)} \quad (6)$$

It is found that the denominator of the SIM of the plant appears in the numerator of DT-PITF. Choosing the discrete-time controller blocks [12] as;

$$A(\epsilon) = \frac{m(\epsilon)}{\lambda(\epsilon)}, B(\epsilon) = \frac{\beta(\epsilon)}{\lambda(\epsilon)}, C(\epsilon) = \frac{\lambda(\epsilon)}{\alpha(\epsilon)} \quad (7)$$

Once this discrete-time PITF is obtained, this must be realized in closed-loop configuration, such as one shown in Fig. 2.

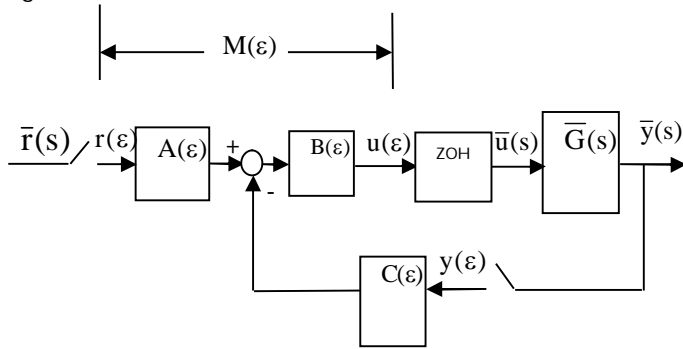


Fig. 2 Discrete-time control system redesigned using the PIM method. And  $\lambda(\epsilon)$  is an arbitrary stable polynomial of appropriate degree [9]. The actual PITF of this control system is given by

$$M(\epsilon) = \frac{m(\epsilon)d_G(\epsilon)}{\beta(\epsilon)n_G(\epsilon) + \alpha(\epsilon)d_G(\epsilon)} \quad (8)$$

The polynomial  $n_G(\epsilon)$  and  $d_G(\epsilon)$  are known from of the plant (see Eq. 2). By equating the target and the actual PITF,

it can be seen that the polynomial  $m(\epsilon)$  must be in the numerator of polynomial  $n_M(\epsilon)$ , the design goal is to choose  $\beta(\epsilon)$ ,  $\alpha(\epsilon)$  in Eq. (7) such that equations (6) and (8) match. For instance, Diophantine equation can be used to achieve this goal by this relation

$$\alpha(\epsilon)d_G(\epsilon) + \beta(\epsilon)n_G(\epsilon) = d_M(\epsilon) \quad (9)$$

under appropriate degree conditions [7]. This can be solved using the eliminant matrix or a state space formulation [7,19].

### 3. SYSTEM MODELING

#### 3.1 Power system model

Fig. 3 shows the one line diagram of the studied system which is a single-machine infinite bus model. The generator is connected to an infinite bus through a transmission line.

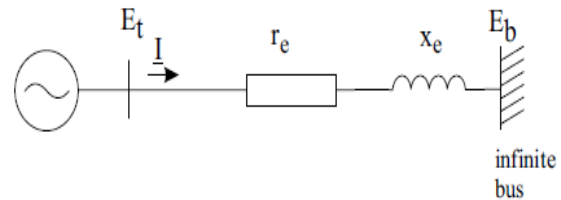


Fig. 3 One machine to infinite bus system.

The block diagram of single machine infinite bus (SMIB) system with controllers is shown in fig. 4. The power system considered in this study is the fourth order linearized one-machine and infinite bus system [21].

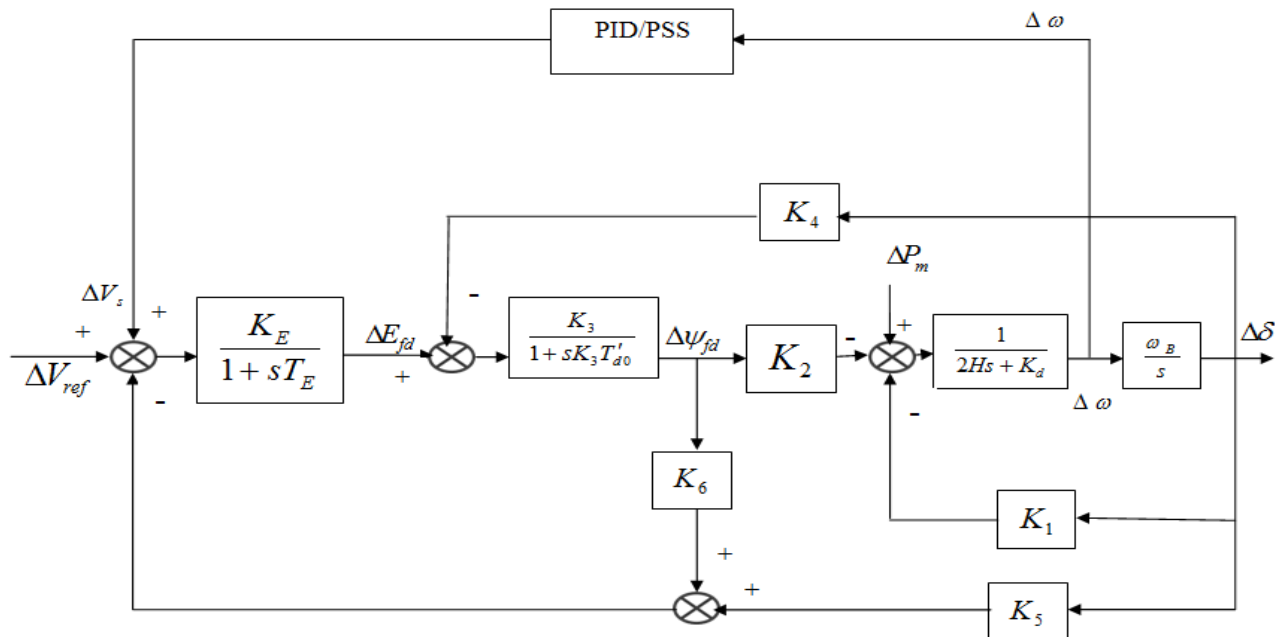


Fig. 4 Block diagram of power system model.

Parameters  $K_1, \dots, K_6$  are the constant of linearized model of synchronous machine,  $K_E$  is the gain of exciter,  $T_E$  is

time constant of exciter,  $T'_{do}$  is the d-axis transient open

circuit time constant, H is the inertia constant and  $K_d$  is damping coefficient. From the block diagram shown in Fig. 4, then the following fourth order linearized one machine infinite bus system can be derived as described in [21]. The equation which describe the excitation system is

$$\frac{\Delta E_{fd}}{V_{ref} - K_5 \Delta \delta - K_6 \Delta \psi_{fd}} = \frac{K_E}{1 + sT_E} \quad (10)$$

This may be written in the standard form

$$\Delta \dot{E}_{fd} = \frac{K_E}{T_E} V_{ref} - \frac{K_5 K_E}{T_E} \Delta \delta - \frac{K_6 K_E}{T_E} \Delta \psi_{fd} - \frac{1}{T_E} \Delta E_{fd} \quad (11)$$

The equation which describe the field circuit is

$$\frac{\Delta \psi_{fd}}{\Delta E_{fd} - K_4 \Delta \delta} = \frac{K_3}{1 + sK_3 T'_{do}} \quad (12)$$

This may be written in the standard form

$$\Delta \dot{\psi}_{fd} = -\frac{K_4}{T'_{do}} \Delta \delta - \frac{1}{K_3 T'_{do}} \Delta \psi_{fd} + \frac{1}{T'_{do}} \Delta E_{fd} \quad (13)$$

The equation which describe the Machine mechanical dynamics loop is

$$\frac{\Delta \omega}{-K_2 \Delta \psi_{fd} + \Delta P_m - K_1 \Delta \delta} = \frac{1}{2Hs + K_d} \quad (14)$$

This may be written in the standard form

$$\Delta \dot{\omega} = -\frac{K_1}{2H} \Delta \delta - \frac{K_2}{2H} \Delta \psi_{fd} - \frac{K_d}{2H} \Delta \omega + \frac{1}{2H} \Delta P_m \quad (15)$$

The equation relating  $\Delta \delta$  to  $\Delta \omega$  is

$$\frac{\Delta \delta}{\Delta \omega} = \frac{\omega_B}{s} \quad (16)$$

This may be written in the standard form

$$\Delta \dot{\delta} = \omega_B \Delta \omega \quad (17)$$

The following fourth order of linearized one machine with infinite bus system. And can be rewritten in the following matrix form.

$$\frac{d}{dt} \begin{bmatrix} \Delta \omega \\ \Delta \delta \\ \Delta \psi_{fd} \\ \Delta E_{fd} \end{bmatrix} = \begin{bmatrix} -\frac{K_d}{2H} & -\frac{K_1}{2H} & -\frac{K_2}{2H} & 0 \\ \omega_B & 0 & 0 & 0 \\ 0 & -\frac{K_4}{T'_{do}} & -\frac{1}{K_3 T'_{do}} & \frac{1}{T'_{do}} \\ 0 & -\frac{K_5 K_E}{T_E} & -\frac{K_6 K_E}{T_E} & -\frac{1}{T_E} \end{bmatrix} \begin{bmatrix} \Delta \omega \\ \Delta \delta \\ \Delta \psi_{fd} \\ \Delta E_{fd} \end{bmatrix} + \begin{bmatrix} \frac{1}{2H} \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_m + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_E}{T_E} \end{bmatrix} \Delta V_{ref} \quad (18)$$

There are two inputs  $\Delta P_m$  and  $\Delta V_{ref}$ , the output is  $\Delta \omega$  in the linearized system. But put  $\Delta V_{ref}$  equal 0. Then single input ( $\Delta P_m$ ), single output ( $\Delta \omega$ ) linearized by the following matrix form.

$$\frac{d}{dt} \begin{bmatrix} \Delta \omega \\ \Delta \delta \\ \Delta \psi_{fd} \\ \Delta E_{fd} \end{bmatrix} = \begin{bmatrix} -\frac{K_d}{2H} & -\frac{K_1}{2H} & -\frac{K_2}{2H} & 0 \\ \omega_B & 0 & 0 & 0 \\ 0 & -\frac{K_4}{T'_{do}} & -\frac{1}{K_3 T'_{do}} & \frac{1}{T'_{do}} \\ 0 & -\frac{K_5 K_E}{T_E} & -\frac{K_6 K_E}{T_E} & -\frac{1}{T_E} \end{bmatrix} \begin{bmatrix} \Delta \omega \\ \Delta \delta \\ \Delta \psi_{fd} \\ \Delta E_{fd} \end{bmatrix} + \begin{bmatrix} \frac{1}{2H} \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_m \quad (19)$$

$$[\Delta \omega] = [1 \ 0 \ 0 \ 0] \begin{bmatrix} \Delta \omega \\ \Delta \delta \\ \Delta \psi_{fd} \\ \Delta E_{fd} \end{bmatrix} + [0] \Delta P_m \quad (20)$$

From the equations (19) and (20), the following fourth order linearized one machine infinite bus system can be given in state variable form as follows:

$$X = AX + BU \quad (21)$$

$$Y = CX + DU$$

The state variables comprise the generator are speed deviation  $\Delta \omega$ , rotor angle deviation  $\Delta \delta$ , transient internal voltage deviation  $\Delta E'_q$  and field voltage deviation  $\Delta E_{fd}$ .

The deviation of the angular velocity  $\Delta \omega$  is assumed to be measured as the output of the system. SMIB Test system data for the small signal stability investigation is taken from the reference [22]. The generator and external network data is given in table (1) and table (2).

TABLE (1) DATA FOR THE SMIB SYSTEM

H	4.63	Xq	0.55
$K_d$	4.4	$K_E$	50.0
$T'_{d0}$	7.67	$T_E$	0.05
$\omega_B$	377.0	$r_e$	0.0
Xd	0.973	$X_e$	0.997
$x'_d$	0.19		

All vales are in per unit to the machine rated MVA and voltage (Kv), and the time constants are in seconds.

TABLE (2) THE K'S

$K_1$	0.5758	$K_4$	0.5266
$K_2$	0.9738	$K_5$	-0.0494
$K_3$	0.6584	$K_6$	0.8450

The values of  $K_1, \dots, K_6$  in the Table (2) are to be calculated according to the operating conditions of the generation system and connected power System. Details of these constants are given in appendix I. Using the data given in Table (1) and Table (2), the transfer function of the power system  $\bar{G}(s)$  given by Fig. 4 and the state space equations given by Eq. 21 can be calculated using the MATLAB function SS2F in the signal processing toolbox and are given by:

$$\bar{G}(s) = \frac{[0.108s^3 + 2.181s^2 + 12.33s]}{[s^4 + 20.67s^3 + 147.2s^2 + 525s + 2876]} \quad (22)$$

The power system transfer function  $\bar{G}(s)$  poles and zero is given in table (3).

TABLE (3) POWER SYSTEM TRANSFER FUNCTION POLES AND ZEROS

poles	Zeros
$-10.2216 \pm j 3.6926$	$-10.0990 \pm j 3.4842$
$-0.1150 \pm j 4.9334$	0.0

### 3.2 Generalized Model of Continuous Time PID Controller

The proportional integral and derivative (PID) controller is widely used in process industries to control the plant (system) for the desired set point. A standard PID controller is known as the "three-term" controller, whose transfer function is generally written in the "parallel form" given by Eq. 23 or the "ideal form" given by Eq. 24.

$$PID(s) = K_p + K_i \frac{1}{s} + K_d s \quad (23)$$

$$PID(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad (24)$$

where  $K_p$  is the proportional gain,  $K_i$  the integral gain,  $K_d$  the derivative gain,  $T_i$  the integral time constant and  $T_d$  the derivative time constant. The "three-term" functionalities are highlighted by the following [23]. The family of PID controllers is constructed from various combinations of the proportional, integral and derivative terms as required to meet specific performance requirements. In the parallel form of the PID controller, three simple gains  $K_p$ ,  $K_i$  and  $K_d$  are used in the decoupled branches of the PID controller [24]. The transfer function of PID controller given by Eq. 23.

$$PID(s) = \frac{K_d s^2 + K_p s + K_i}{s} = \frac{N(s)}{D(s)} \quad (25)$$

This transfer function is non-proper and is therefore difficult to realise in practice. The proper transfer function must be: order  $D(s) \geq$  order  $N(s)$ . These are generally easier to realise, and also reduce the susceptibility of the derivative action to noise. Then the Practical PID controllers become as shown in Fig. 5.

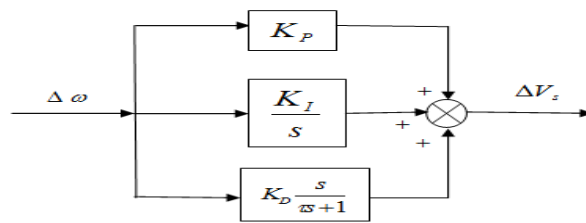


Fig. 5 Modified PID controller structure.

The deviation of the angular velocity  $\Delta\omega$  is assumed to be measured as the output of the system which is controlled by PID controller. Then the transfer function of the modified PID controller become as

$$PID(s) = K_p + K_i \frac{1}{s} + K_d \frac{s}{\tau s + 1} \quad (26)$$

The term  $\left(\frac{1}{\tau s + 1}\right)$  acts as an effective low-pass filter on the

D regulator to attenuate noise in the derivative block. If  $\tau = 0$  the original PID form is obtained. Typically  $\tau = 0.01$  to place the filter as far away from the derivative action as possible. It is generally impractical to move the filter further away as the control action becomes too much and can-not be realised. PID controller parameters are determined from the Ziegler-Nichols tuning given in table (4).

TABLE (4) PID CONTROLLER PARAMETERS

KP	15.5
KI	5.0
KD	0.0115

Utilizing the parameter of the PID controller, the transfer function of the PID controller given can be calculated as;

$$PID(s) = \frac{a_2 s^2 + a_1 s + a_0}{b_2 s^2 + b_1 s + b_0} = \frac{0.1665 s^2 + 15.55 s + 5}{0.01 s^2 + s} \quad (27)$$

The PID controller transfer function  $PID(s)$  poles and zero is given in table (5).

TABLE (5) PID CONTROLLER POLES AND ZEROS

Poles	zeros
-100	-0.930707
0	-0.3227

### 3.3 Power System Stabilizer Modeling

The continuous time PSS type is widely used in the power system to improve the damping oscillations of the power system; sometime it is called the damping controller. Because the power system is very oscillatory, the objective of the PSS is to enhance the damping force and necessarily to improve the dynamical stability of the power system [22]. The PSS consists of a phase compensation block, a signal washout block, and a gain block as shown in Fig. 6.

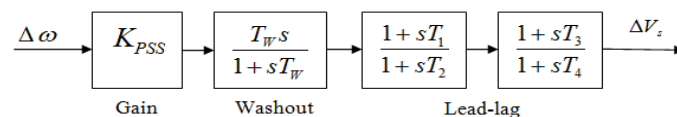


Fig. 6 Block diagram of Power System Stabilizer.

The transfer function of a continuous-time, lead-lag type, power system stabilizer is given by

$$PSS(s) = \left( \frac{k_{PSS} T_W s}{1 + sT_W} \right) \left( \frac{1 + sT_1}{1 + sT_2} \right) \left( \frac{1 + sT_3}{1 + sT_4} \right) \quad (28)$$

The gain  $k_{PSS}$  is chosen by trial and error method and the washout time constant  $T_W$  is chosen in between 0 to 20 sec. The washout stage is used to prevent a steady-state voltage shift;  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  are time constants of the two phase-lead stages. The parameters of PSS are tuned by trial and error method, so as to achieve the desired damping ratio of the electromechanical mode and compensate for the phase shift between the control signal and the resulting electrical power deviation. The parameters of PSS are given in table (6).

TABLE (6) PSS PARAMETERS

$k_{PSS}$	20.0	$T_1$	0.15	$T_3$	0.05
$T_W$	10	$T_2$	0.05	$T_4$	0.15

Utilizing the data of the PSS given, the transfer function of the PSS described by Eq. 28 can be calculated as follows [25].

$$PSS(s) = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_3 s^3 + b_2 s^2 + b_1 s + b_0} = \frac{1.5s^3 + 40s^2 + 200s}{0.075s^3 + 2.007s^2 + 10.2s + 1} \quad (29)$$

The PSS transfer function  $PSS(s)$  poles and zero is given in table (7).

TABLE (7) PSS POLES AND ZEROS

Poles	zeros
-20.000	0
-6.6667	-20.0000
-0.1000	-6.6667

#### 4. APPLICATION OF TRADITIONAL DISCRETIZATION METHOD (TUSTIN'S METHOD) TO POWER SYSTEM MODEL

Discretization of analog controllers by using bilinear method (Tustin's method) is investigated [26]. By replacing each S-domain in analog controllers to Z-domain, according to this relation.

$$\left( s = \frac{z-1}{\frac{T}{2}(z+1)} \right), \text{ Where T is sampling time} \quad (30)$$

Then, the transfer function of a digital PID controller (Tustin's method) is

$$PID(z) = \frac{\left( a_2 + \frac{T}{2} a_1 + a_0 \frac{T^2}{4} \right) z^2 + \left( -2a_2 + a_0 \frac{T^2}{2} \right) z + \left( a_2 - \frac{T}{2} a_1 + a_0 \frac{T^2}{4} \right)}{\left( b_2 + \frac{T}{2} b_1 \right) z^2 - 2b_2 z + \left( b_2 - \frac{T}{2} b_1 \right)} \quad (31)$$

And the transfer function of a digital PSS (Tustin's method) is

$$PSS(z) = \frac{\left[ \left( a_3 + \frac{T}{2} a_2 + \frac{T^2}{4} a_1 \right) z^3 + \left( -3a_3 - \frac{T}{2} a_2 + \frac{T^2}{4} a_1 \right) z^2 + \left( 3a_3 - \frac{T}{2} a_2 - a_1 \frac{T^2}{4} \right) z + \left( -a_3 + \frac{T}{2} a_2 - \frac{T^2}{4} a_1 \right) \right]}{\left( b_3 + \frac{T}{2} b_2 + \frac{T^2}{4} b_1 + \frac{T^3}{8} b_0 \right) z^3 + \left( -3b_3 - \frac{T}{2} b_2 + \frac{T^2}{4} b_1 + \frac{3T^3}{8} b_0 \right) z^2 + \left( 3b_3 - \frac{T}{2} b_2 - \frac{T^2}{4} b_1 + \frac{3T^3}{8} b_0 \right) z + \left( -b_3 + \frac{T}{2} b_2 - \frac{T^2}{4} b_1 + \frac{T^3}{8} b_0 \right)} \quad (32)$$

After design of discrete-time PID, PSS controllers for discrete-time control systems by using traditional method (Tustin's method) compare it with design of discrete-time control system by using the proposed method (PIM) which presented in section II.

#### 5. APPLICATION OF PROPOSED DISCRETIZATION METHOD (PIM METHOD) TO POWER SYSTEM MODEL

First design a suitable analog control system, and then discretized it by the Plant Input Mapping (PIM) method. The transfer function  $G(s)$  for the power system given by Eq. 22, the analog controller is placed on the block  $B(s)$  of Fig. 1 with the blocks  $A(s)$  and  $C(s)$  equal to 1. Simulations responses of the power system based on the linear model given by Eq. 21 are presented. The power system is subject to a step change in the mechanical torque denoted by  $\Delta P_m$ , the signal to be controlled is the rotor speed denoted by  $\Delta \omega$ . The three controller blocks,  $A(\varepsilon)$ ,  $C(\varepsilon)$  and  $B(\varepsilon)$  are calculated using the procedure of design PIM method which presented in section II.

#### 6. SYSTEM RESPONSE AND ANALYSIS

The test system has been modeled through Matlab programming. Fig. 7 show the simulations result of the analog PID, PSS controllers. The performance of the continuous-time PSS converge to the continuous-time PID controller. Fig. 8 to Fig. 15 show the responses and the plant input obtained using the control rates of 5Hz, 3.33Hz, 2.5Hz, and 2.12Hz, respectively. Each controller is designed with the following control specifications in mind:

- Overshoot is less than 20% of the amplitude of the reference step signal.
- Settling time is faster than 10 sec.
- Steady state error is smaller than 0.5 degree.

Fig. 8 to Fig. 11 show simulations results of the traditional digital design technique Tustin's method. It is noticed that the Tustin's controllers are stable for small sampling rates. On other hand, it is found that Tustin's method for both PID, PSS controllers is violated when sampling interval becomes large.

At the 5Hz and 3.33Hz control rate, the performance of Tustin's PID controller almost matched with Tustin's PSS but it produces an overshoot than Tustin's PSS. At the 2.5Hz control rate, the Tustin's response of both controllers oscillates and is not satisfactory but settle even after 8 sec. The Tustin's response of both PID, PSS controllers becomes unstable and oscillates violently to such an extent that it is

not acceptable and doesn't settle even after 10 sec at 2.12Hz control rate.

Fig. 12 to Fig. 15 show simulations results of the proposed digital redesign technique PIM method. It is noticed that the proposed algorithm is stable for any sampling rate, as well as it takes the closed loop characteristic into consideration.

At the 5Hz control rate, the response of PIM-PID controller produces a smaller overshoot than PIM-PSS (Overshoot is almost less than 10%), while both PIM-controllers are closely match to the analog case in Fig. 8. At the 3.33Hz control rate, the overshoot of the PIM-PID controller become larger than the corresponding case of 5Hz, PIM-PSS is settling faster than PIM-PID. It is noted from Fig. 14 and Fig. 15 that when the sampling rate becomes large the proposed digital redesign technique PIM of PSS guarantees stability, be convergent to CT-PSS and it settles in the same time as the analog one with no steady state error and almost no oscillation but the PIM of PID controller produces a different transient response from analog one and it has a small overshoot.

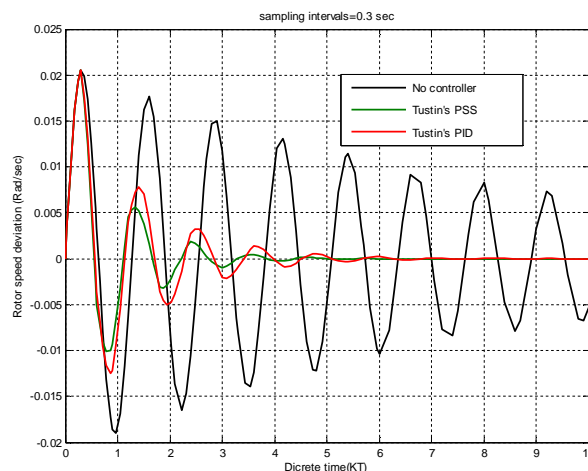


Fig. 9 Dynamic responses to step change in the mechanical torque in the presence of Tustin's PSS and Tustin's PID (sampling interval 0.3 s)

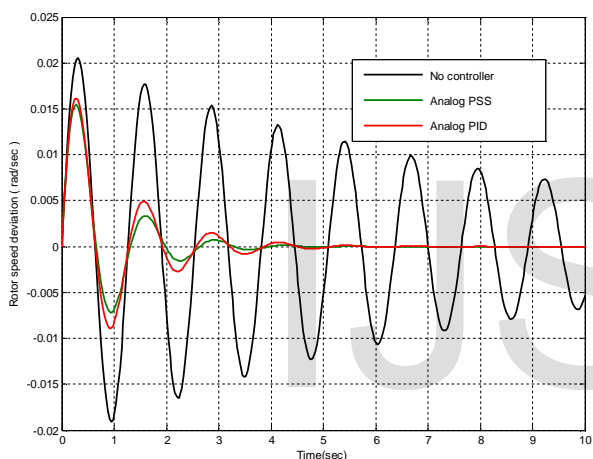


Fig. 7 Dynamic responses to step change in the mechanical torque.

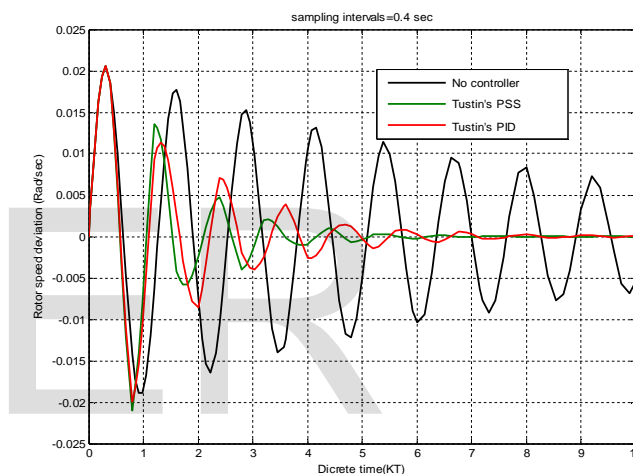


Fig. 10 Dynamic responses to step change in the mechanical torque in the presence of Tustin's PSS and Tustin's PID (sampling interval 0.4 s)

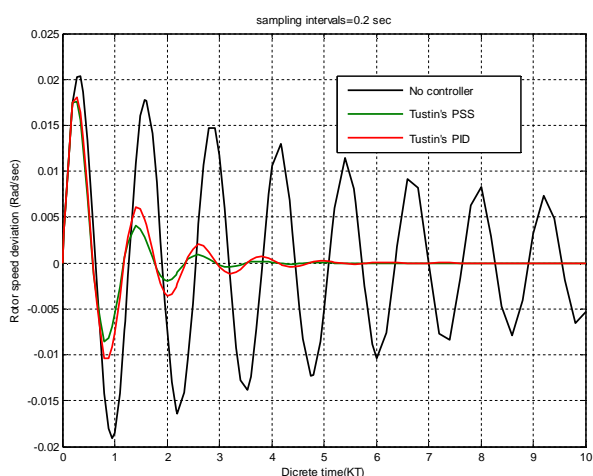


Fig. 8 Dynamic responses to step change in the mechanical torque in the presence of Tustin's PSS and Tustin's PID (sampling interval 0.2 s)

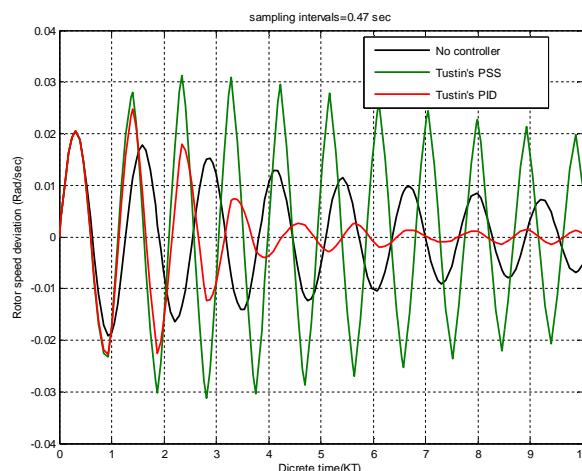


Fig. 11 Dynamic responses to step change in the mechanical torque in the presence of Tustin's PSS and Tustin's PID (sampling interval 0.47 s)

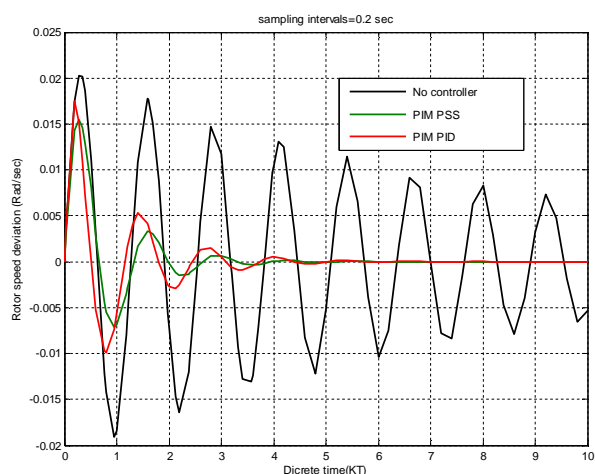


Fig. 12 Dynamic responses to step change in the mechanical torque in the presence of PIM-PSS and PIM-PID (sampling interval 0.2 s)

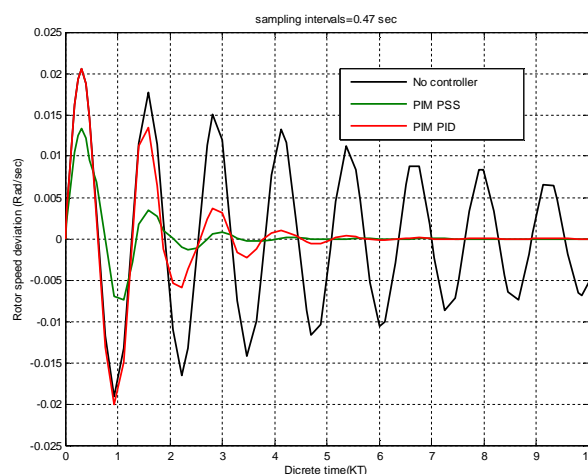


Fig. 15 Dynamic responses to step change in the mechanical torque in the presence of PIM-PSS and PIM-PID (sampling interval 0.47 s)

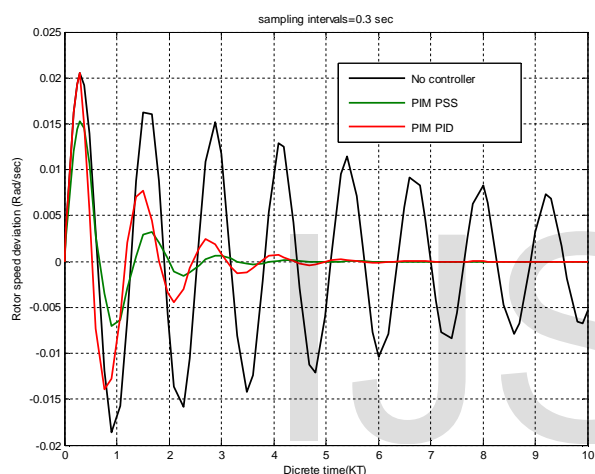


Fig. 13 Dynamic responses to step change in the mechanical torque in the presence of PIM-PSS and PIM-PID (sampling interval 0.3 s)

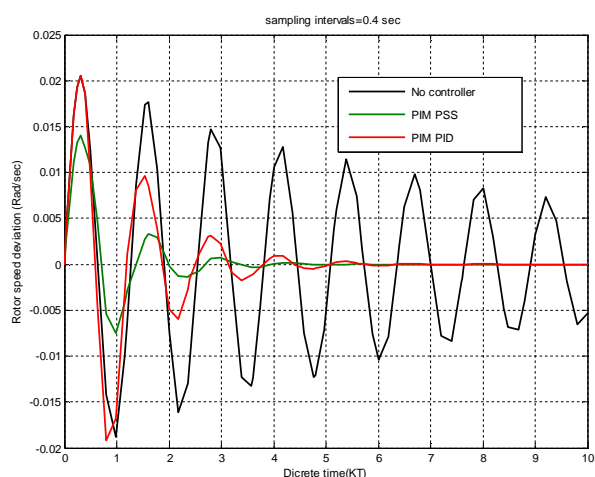


Fig. 14 Dynamic responses to step change in the mechanical torque in the presence of PIM-PSS and PIM-PID (sampling interval 0.4 s)

### 7. CONCLUSION

From the results obtained for different approaches to designing discrete-time PID, PSS controllers for a continuous-time system in a feedback configuration which applied to a single machine infinite power system for stability enhancement. It can be concluded that the proposed algorithm (PIM method) is stable for any sampling rates, while the traditional discretization method (Tustin's method) is produce satisfactory results only; when the sampling period is sufficiently low.

The results observed by simulations showed that

- a) The performance of Tustin's PID controller almost matched with Tustin's PSS but it produces an overshoot than Tustin's PSS at small sampling rates. Tustin's controllers are unstable and oscillate violently when sampling interval becomes large.
- b) Both PIM-controllers are stable for any sampling rates and closely match to the analog case, while the PIM-PSS is effective in the improvement of settling time but the PIM of PID controller produces a different transient response from analog one and it has a small overshoot.

### APPENDIX I

The constants  $K_1, \dots, K_6$  are evaluated with transmission line resistance  $r_e=0$  and are given as follows:

$$K_1 = \frac{X_q - X_d}{(X_e + X'_d)} I_{q0} V_0 \sin \delta_0 + \frac{E_{q0} V_0 \cos \delta_0}{(X_e + X_q)}$$

$$K_2 = \frac{V_0 \sin \delta_0}{(X_e + X'_d)}$$

$$K_3 = \frac{X'_d + X_e}{(X_d + X_e)}$$



$$K_4 = \frac{X_d - X'_d}{(X_e + X'_d)} V_0 \sin \delta_0$$

$$k_5 = \frac{X_q}{(X_e + X_q)} \frac{V_{d0}}{V_{t0}} V_0 \cos \delta_0 + \frac{X'_d}{(X_e + X'_d)} \frac{V_{q0}}{V_{t0}} V_0 \sin \delta_0$$

$$K_6 = \frac{X_e}{(X_e + X'_d)} \frac{V_{q0}}{V_{t0}}$$

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